

Algebra 1: Problem Set 8B

Name _____

Date _____ Period _____

SCIENTIFIC NOTATION

Scientific notation is based on powers of 10. Scientific notation allows us to represent, manipulate, and compute very large and very small numbers.

Numbers in scientific notation are written as $a \times 10^n$ where a is a decimal number greater than or equal to 1 and less than 10 and n is an integer (positive or negative). $1 \leq a < 10$

Convert scientific notation to decimal:

$$5.1 \times 10^4 = 51,000$$

$$3.754 \times 10^{-7} = 0.0000003754$$

$$5 \times 10^{12} = 5,000,000,000,000$$

Convert decimal to scientific notation:

$$47,800,000,000 = 4.78 \times 10^{10}$$

(move decimal point 10 places to the left)

$$0.00000023 = 2.3 \times 10^{-7}$$

(move decimal point 7 places to the right)

We can multiply and divide with scientific notation using the very same rules of Powers and Exponents we have just learned:

Example A)

$$\begin{aligned} & (3.4 \times 10^4)(8.2 \times 10^6) \\ &= (3.4 \cdot 8.2) \times (10^4 \cdot 10^6) && \text{re-group/re-order!!} \\ &= (27.88) \times (10^{10}) && \text{simplify} \\ &= 2.788 \times 10^{11} && \text{re-write in scientific notation} \end{aligned}$$

Example B)

$$\begin{aligned} & \frac{1.2 \times 10^{-1}}{4.8 \times 10^{-4}} \\ &= \frac{1.2}{4.8} \times \frac{10^{-1}}{10^{-4}} && \text{re-write as a product of fractions} \\ &= 0.25 \times 10^3 && \text{simplify} \\ &= 2.5 \times 10^2 && \text{re-write in scientific notation} \end{aligned}$$

Why use scientific notation??

For example:

Mercury's mean distance from the sun is 5.79×10^7 kilometers (km)

Jupiter's's mean distance from the sun is 7.78×10^8 km

Question: Percentage-wise, how far is Mercury from the sun compared to Jupiter?

So, all we have to do is divide: $\frac{5.79 \times 10^7}{7.78 \times 10^8}$

Hmmm... that doesn't really fit into every calculator, does it?

So, we can use scientific notation!!

$$\frac{5.79 \times 10^7}{7.78 \times 10^8} = \frac{5.79}{7.78} \times \frac{10^7}{10^8} \approx 0.74 \times 10^{-1} = 0.74 \left(\frac{1}{10}\right) = 0.074 \text{ or } 7.4\%$$

- 1) *Re-write the problems below using scientific notation*
- 2) *Compute the solution*
- 3) *Write the solution in both scientific notation and decimal notation*

1) $7,000,000 \cdot 420,000$

2)
$$\frac{6,480,000,000}{4,320,000}$$

Answer :

$$\begin{aligned} &7 \times 10^6 \cdot 4.2 \times 10^5 \\ &(7)(4.2) \times 10^6 \cdot 10^5 \\ &29.4 \times 10^{11} \\ &2.94 \times 10^{12} \\ &2,940,000,000,000 \end{aligned}$$

Answer :

$$\begin{aligned} &6.48 \times 10^9 \div 4.32 \times 10^6 \\ &\frac{6.48}{4.32} \cdot \frac{10^9}{10^6} \\ &1.5 \times 10^3 \\ &1,500 \end{aligned}$$

INCONTROVERTIBLE TRUTHS

$$a^{-3} = \frac{1}{a^3}$$

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

$$a^0 = 1$$

$$\frac{1}{a^{-5}} = a^5$$

$$a^2 \cdot a^3 = a^{2+3} = a^5$$

$$\frac{a^5}{a^3} = a^{5-3} = a^2$$

$$(a^2)^3 = a^{2 \cdot 3} = a^6$$

Simplify the following expressions using **positive exponents**.

1. 3^{-5}

$$\text{Ans} = \frac{1}{3^5} = \frac{1}{243}$$

2. $3x^{-6}y^4$

$$\text{Ans} = \frac{3y^4}{x^6}$$

3. $\frac{4}{5y^{-4}}$

$$\text{Ans} = \frac{4y^4}{5}$$

4. $\frac{6x^{-5}y^5}{9x^3y^{-3}}$

$$\text{Ans} = \frac{2y^8}{3x^8}$$

5. $(12)^{-6} \cdot (12)^5$

$$\text{Ans} = 12^{-6+5} = 12^{-1} = \frac{1}{12}$$

6. $\left(\frac{3x^{-2}}{4y^{-3}}\right)^{-3}$ "power to a power"

$$\left(\frac{3x^{-2}}{4y^{-3}}\right)^{-3} = \frac{3^{-3}x^6}{4^{-3}y^9} = \frac{64x^6}{27y^9}$$

7. $\left(\frac{2x^2y^{-4}}{3x^{-4}y^3}\right)^2$

Answer:

$$\frac{(2x^2y^{-4})^2}{(3x^{-4}y^3)^2} = \frac{2^2x^4y^{-8}}{3^2x^{-8}y^6} = \frac{4x^{12}}{9y^{14}}$$

8. $\frac{4x^2y^4}{8xy^6} \div \frac{2xy^2}{8x^3y^3}$

Answer:

$$\frac{4x^2y^4}{8xy^6} \cdot \frac{8x^3y^3}{2xy^2} = \frac{2x^3}{y}$$

Complete the INCONTROVERTIBLE TRUTHS below

$a^{-3} =$	$\left(\frac{a}{b}\right)^3 =$	$a^0 =$	$\frac{1}{a^{-5}} =$
$a^2 \cdot a^3 = a^{\quad} =$	$\frac{a^5}{a^3} = a^{\quad} =$	$(a^2)^3 = a^{\quad} =$	

Simplify the following expressions using **positive exponents**.

9. $(4.1 \times 10^{-3})(3.2 \times 10^9)$

Answer

$$(4.1)(3.2) \times 10^{-3} \times 10^9$$

$$13.12 \times 10^6$$

$$1.312 \times 10^7$$

10. $\frac{9.3 \times 10^7}{3.1 \times 10^3}$

$$3.0 \times 10^4$$

11. $\left(\frac{4m^{-3}n^4}{9m^4n^{-2}}\right)^2$

Answer:

$$\frac{(4m^{-3}n^4)^2}{(9m^4n^{-2})^2} = \frac{4^2 m^{-6} n^8}{9^2 m^8 n^{-4}} = \frac{16n^{12}}{81m^{14}}$$

12. $\frac{6x^{-3}y^4}{9x^2y^6} \div \frac{18x^2y^{-2}}{12x^{-3}y^3}$

See below:

Solution for #12:

$$\frac{6x^{-3}y^4}{9x^2y^6} \div \frac{18x^2y^{-2}}{12x^{-3}y^3} = \frac{6x^{-3}y^4}{9x^2y^6} \cdot \frac{12x^{-3}y^3}{18x^2y^{-2}} = \frac{4x^{-6}y^7}{9x^4y^4} = \frac{4y^3}{9x^{10}}$$

Alternate solution method for #6: $\left(\frac{3y^3}{4x^2}\right)^{-3} = \frac{1}{\left(\frac{3y^3}{4x^2}\right)^3} = \frac{1}{\frac{(3y^3)^3}{(4x^2)^3}} = \frac{64x^6}{27y^9}$ WHAT FUN!!